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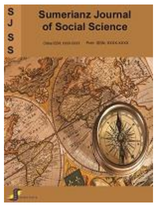

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Deductive Reasoning Based on the Valid Aristotelian Syllogism *AEE-2*

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Abstract

This paper firstly formalizes Aristotelian syllogisms based on the tripartite structure of categorical propositions, and then uses the truth definitions of categorical propositions to prove the validity of the Aristotelian syllogism *AEE-2*. Then, the remaining 23 valid syllogisms are derived from the syllogism *AEE-2* with the help of relevant facts, inner and outer negation definitions of quantifiers, and deductive rules. In other words, this paper reveals the reducible relationship between/among these 24 syllogisms and establishes a succinct formal reason system for Aristotelian syllogistic. The deductive reasoning not only ensures consistency in its results, but also provides a concise mathematical paradigm for other types of syllogisms.

Keywords: Aristotelian syllogisms; Deductive reasoning; Reducible relationship; Validity.

1. Introduction

In natural language, there are many types of syllogisms, such as Aristotelian syllogisms (Yijiang, 2023), modal syllogisms (Cheng, 2023), generalized syllogisms (Moss, 2010), and so on, which are common and important forms of reasoning in social life and logic (Jing and Xiaojun, 2023), and has been widely studied since Aristotle (Murinová and Novák, 2012). This paper focuses on the study of Aristotelian syllogisms. And in the following, unless otherwise specified, syllogisms refer to Aristotelian syllogisms.

Aristotelian syllogisms involve the following four types of statements: *all r*s are *t*, *no r*s are *t*, *some r*s are *t*, and *not all r*s are *t*. They are abbreviated as Propositions *A*, *E*, *I*, and *O* respectively, where *all*, *no*, *some*, and *not all* are called Aristotelian quantifiers (Long and Xiaojun, 2023). It is known that there are only 24 Aristotelian syllogisms out of 256 ones (Xiaojun *et al.*, 2022).

Łukasiewicz (1957), took the syllogisms *AAA-1* and *AII-3* as basic axioms to derive the other 22 valid ones. Xiaojun and Sheng (2016), deduced the other 22 valid syllogisms on the basis of the two syllogisms *AAA-1* and *EAE-1*. This paper only uses the syllogism *AEE-2* as a basic axiom reasoning basis to infer the remaining 23 valid ones.

2. Formal Aristotelian Syllogistic

In order to construct the formal system, the following four parts need to be provided: (1) primitive symbols; (2) formation rules of well-formed formulas (abbreviated as wff); (3) basic axioms; (4) rules of inference. In the following, let α, β, γ and δ be wffs.

2.1. Primitive Symbols

- lexical variables: *r*, *s*, *t*
- negative operator: ¬

- implication operator: $\rightarrow$
- quantifier: *all*
- brackets: (,)

2.2. Formation Rules

- If Q is a quantifier, r and t are lexical variables, then $Q(r, t)$ is a wff.
- If α and β are wffs, then $\neg\alpha$ and $\alpha\rightarrow\beta$ are wffs.
- Only the formulas obtained based on (2.2.1) and (2.2.2) are wffs.

For instance, $all(r, t)$, and $all(r, t)\rightarrow some(r, s)$ are wffs that can be seen as '*all* rs are t' and 'if *all* rs are t, then *some* rs are s', respectively. The others are similar. It can be known that Aristotelian syllogisms contain the following four types of categorical propositions, namely $all(r, t)$, $no(r, t)$, $some(r, t)$, and $not\ all(r, t)$. They are respectively abbreviated as Propositions A, E, I and O.

2.3. Basic Axioms

A1: If α is a valid formula in first-order logic, then $\vdash\alpha$.

A2: $\vdash all(t, s)\wedge no(r, s)\rightarrow no(r, t)$(that is, the syllogism *AEE-2*).

In these axioms, '$\vdash\alpha$' means that α is provable. The other cases are similar.

2.4. Rules of Inference

R1: (subsequent weakening): From$\vdash(\alpha\wedge\beta\rightarrow\gamma)$ and $\vdash(\gamma\rightarrow\delta)$, infer $\vdash(\alpha\wedge\beta\rightarrow\delta)$.

R2: (anti-syllogism): From $\vdash(\alpha\wedge\beta\rightarrow\gamma)$, infer $\vdash(\neg\gamma\wedge\alpha\rightarrow\neg\beta)$.

2.5. Related Definitions

D1 (biconditional $\leftrightarrow$) : $(\alpha\leftrightarrow\beta)=_{def}(\alpha\rightarrow\beta)\wedge(\beta\rightarrow\alpha)$.

D2: $(\alpha\wedge\beta)=_{def}\neg(\alpha\rightarrow\neg\beta)$.

D3 (inner negative quantifier): $Q\neg(r, t) =_{def} Q(r, D-t)$.

D4 (outer negative quantifier): $(\neg Q)(r, t) =_{def}$ It is not that $Q(r, t)$.

D5 (true value of quantifier): $all(r, t)=_{def} R\subseteq T$.

D6 (true value of quantifier): $some(r, t)=_{def} R\cap T\neq\varnothing$.

D7 (true value of quantifier): $no(r, t)=_{def} R\cap T=\varnothing$.

D8 (true value of quantifier): $not\ all(r, t)=_{def} R\not\subseteq T$.

2.6. Related Facts

Fact 1 (inner negation):

F1: $\vdash all(r, t)\leftrightarrow no\neg\ (r, t)$; F2: $\vdash no(r, t)\leftrightarrow all\neg\ (r, t)$;

F3: $\vdash some(r, t)\leftrightarrow not\ all\neg\ (r, t)$; F4: $\vdash not\ all(r, t)\leftrightarrow some\neg\ (r, t)$.

Fact 2 (outer negation):

F5: $\vdash all(r, t) \leftrightarrow \neg not\ all(r, t)$; F6: $\vdash not\ all(r, t) \leftrightarrow\neg all(r, t)$;

F7: $\vdash some(r, t) \leftrightarrow\neg no(r, t)$; F8: $\vdash no(r, t) \leftrightarrow\neg some(r, t)$.

Fact 3 (symmetry of *some* and *no*):

F9: $\vdash some(r, t) \leftrightarrow some(t, r)$; F10: $\vdash no(r, t) \leftrightarrow no(t, r)$.

Fact 4 (assertoric subalternations):

F11: $\vdash no(r, t) \rightarrow not\ all(r, t)$; F12: $\vdash all(r, t) \rightarrow some(r, t)$.

3. Reducible Relationship between the Other 23 Valid Syllogisms and the Syllogism *AEE-2*

The following Theorem 1 shows the syllogism *AEE-2* is valid. '*AEE-2*AEE-2→AEE-4 in Theorem 2 indicates that the validity of the syllogism *AEE-4*can be derived from the validity of the syllogism *AEE-2*. That is to say, there is a reducible relationship between these two syllogisms. The others are similar.

Theorem 1 (*AEE-2*)**:** $\vdash all(t, s)\wedge no(r, s)\rightarrow no(r, t)$ is valid.

Proof: Suppose that $all(t, s)$ and $no(r, s)$ are true, then $T\subseteq S$ and $R\cap S=\varnothing$ are true according to Definition 5 and 7, respectively. Hence, it can be seen that $R\cap T=\varnothing$ is true. Thus $no(r, t)$ is true in line with Definition 7. It follows that $\vdash all(t, s)\wedge no(r, s)\rightarrow no(r, t)$ is valid, as required.

Theorem 2: The remaining 23 valid syllogisms can be deduced just from the syllogism *AEE-2*:

(1) ⊢AEE-2→AEE-4

(2) ⊢AEE-2→AEE-4→EAE-1

(3) ⊢AEE-2→EAE-2

(4) ⊢AEE-2→AII-1

(5) ⊢AEE-2→AII-1→AII-3

(6) ⊢AEE-2→AII-1→AII-3→IAI-3

(7) $\vdash$AEE-2$\rightarrow$AII-1$\rightarrow$IAI-4
(8) $\vdash$AEE-2$\rightarrow$AEO-2
(9) $\vdash$AEE-2$\rightarrow$AEO-2$\rightarrow$AEO-4
(10) $\vdash$AEE-2$\rightarrow$AEE-4$\rightarrow$EAE-1$\rightarrow$EAO-1
(11) $\vdash$AEE-2$\rightarrow$AEE-4$\rightarrow$EAE-1$\rightarrow$EAO-1$\rightarrow$EAO-2
(12) $\vdash$AEE-2$\rightarrow$AEE-4$\rightarrow$EAE-1$\rightarrow$EAO-1$\rightarrow$EAO-2$\rightarrow$AAI-3
(13) $\vdash$AEE-2$\rightarrow$AEO-2$\rightarrow$AAI-1
(14) $\vdash$AEE-2$\rightarrow$AEO-2$\rightarrow$AAI-1$\rightarrow$AAI-4
(15) $\vdash$AEE-2$\rightarrow$AEO-2$\rightarrow$AAI-1$\rightarrow$AAI-4$\rightarrow$EAO-4
(16) $\vdash$AEE-2$\rightarrow$AEO-2$\rightarrow$AAI-1$\rightarrow$AAI-4$\rightarrow$EAO-4$\rightarrow$EAO-3
(17) $\vdash$AEE-2$\rightarrow$AEE-4$\rightarrow$EAE-1$\rightarrow$AAA-1
(18) $\vdash$AEE-2$\rightarrow$AEE-4$\rightarrow$EAE-1$\rightarrow$AAA-1$\rightarrow$OAO-3
(19) $\vdash$AEE-2$\rightarrow$AEE-4$\rightarrow$EAE-1$\rightarrow$AAA-1$\rightarrow$OAO-3$\rightarrow$AOO-2
(20) $\vdash$AEE-2$\rightarrow$AII-1$\rightarrow$EIO-1
(21) $\vdash$AEE-2$\rightarrow$AII-1$\rightarrow$EIO-1$\rightarrow$EIO-3
(22) $\vdash$AEE-2$\rightarrow$AII-1$\rightarrow$EIO-1$\rightarrow$EIO-3$\rightarrow$EIO-4
(23) $\vdash$AEE-2$\rightarrow$AII-1$\rightarrow$EIO-1$\rightarrow$EIO-2

Proof:
[1] $\vdash \mathit{all}(t, s)\wedge \mathit{no}(r, s) \rightarrow \mathit{no}(r, t)$ (i.e. *AEE-2*, basic axiom A2)
[2] $\vdash \mathit{all}(t, s)\wedge \mathit{no}(s, r) \rightarrow \mathit{no}(r, t)$ (i.e. *AEE-4*, by [1] and F10)
[3] $\vdash \mathit{all}(t, s)\wedge \mathit{no}(s, r) \rightarrow \mathit{no}(t, r)$ (i.e. *EAE-1*, by [2] and F10)
[4] $\vdash \mathit{all}(t, s)\wedge \mathit{no}(r, s) \rightarrow \mathit{no}(t, r)$ (i.e. *EAE-2*, by [1] and F10)
[5] $\vdash \neg \mathit{no}(r, t)\wedge \mathit{all}(t, s) \rightarrow \neg \mathit{no}(r, s)$ (by [1] and R2)
[6] $\vdash \mathit{some}(r, t)\wedge \mathit{all}(t, s) \rightarrow \mathit{some}(r, s)$ (i.e. *AII-1*, by [5] and F7)
[7] $\vdash \mathit{some}(t, r)\wedge \mathit{all}(t, s) \rightarrow \mathit{some}(r, s)$ (i.e. *AII-3*, by [6] and F9)
[8] $\vdash \mathit{some}(t, r)\wedge \mathit{all}(t, s) \rightarrow \mathit{some}(s, r)$ (i.e. *IAI-3*, by [7] and F9)
[9] $\vdash \mathit{some}(r, t)\wedge \mathit{all}(t, s) \rightarrow \mathit{some}(s, r)$ (i.e. *IAI-4*, by [6] and F9)
[10] $\vdash \mathit{no}(r, t) \rightarrow \mathit{not\ all}(r, t)$ (by F11)
[11] $\vdash \mathit{all}(t, s)\wedge \mathit{no}(r, s) \rightarrow \mathit{not\ all}(r, t)$ (i.e. *AEO-2*, by [1], [10] and R1)
[12] $\vdash \mathit{all}(t, s)\wedge \mathit{no}(s, r) \rightarrow \mathit{not\ all}(r, t)$ (i.e. *AEO-4*, by [11] and F10)
[13] $\vdash \mathit{all}(t, s)\wedge \mathit{no}(s, r) \rightarrow \mathit{not\ all}(t, r)$ (i.e. *EAO-1*, by [3], [10] and R1)
[14] $\vdash \mathit{all}(t, s)\wedge \mathit{no}(r, s) \rightarrow \mathit{not\ all}(t, r)$ (i.e. *EAO-2*, by [13] and F10)
[15] $\vdash \neg \mathit{not\ all}(t, r)\wedge \mathit{all}(t, s) \rightarrow \neg \mathit{no}(r, s)$ (by [14] and R2)
[16] $\vdash \mathit{all}(t, r)\wedge \mathit{all}(t, s) \rightarrow \mathit{some}(r, s)$ (i.e. *AAI-3*, by [15], F5 and F7)
[17] $\vdash \neg \mathit{not\ all}(r, t)\wedge \mathit{all}(t, s) \rightarrow \neg \mathit{no}(r, s)$ (by [11] and R2)
[18] $\vdash \mathit{all}(r, t)\wedge \mathit{all}(t, s) \rightarrow \mathit{some}(r, s)$ (i.e. *AAI-1*, by [17], F5 and F7)
[19] $\vdash \mathit{all}(r, t)\wedge \mathit{all}(t, s) \rightarrow \mathit{some}(s, r)$ (i.e. *AAI-4*, by [18] and F9)
[20] $\vdash \neg \mathit{some}(s, r)\wedge \mathit{all}(r, t) \rightarrow \neg \mathit{all}(t, s)$ (by [19] and R2)
[21] $\vdash \mathit{no}(s, r)\wedge \mathit{all}(r, t) \rightarrow \mathit{not\ all}(t, s)$ (i.e. *EAO-4*, by [20], F8 and F6)
[22] $\vdash \mathit{no}(r, s)\wedge \mathit{all}(r, t) \rightarrow \mathit{not\ all}(t, s)$ (i.e. *EAO-3*, by [21] and F10)
[23] $\vdash \mathit{all}(t, s)\wedge \mathit{all}\neg(s, r) \rightarrow \mathit{all}\neg(t, r)$ (by [3] and F2)
[24] $\vdash \mathit{all}(t, s)\wedge \mathit{all}(s, D-r) \rightarrow \mathit{all}(t, D-r)$ (i.e.*AAA-1*,by [23] and D3)
[25] $\vdash \neg \mathit{all}(t, D-r)\wedge \mathit{all}(t, s) \rightarrow \neg \mathit{all}(s, D-r)$ (by [24] and R2)
[26] $\vdash \mathit{not\ all}(t, D-r)\wedge \mathit{all}(t, s) \rightarrow \mathit{not\ all}(s, D-r)$ (i.e. *OAO-3*, by [25] and F6)
[27] $\vdash \neg \mathit{not\ all}(s, D-r)\wedge \mathit{not\ all}(t, D-r) \rightarrow \neg \mathit{all}(t, s)$ (by [26] and R2)
[28] $\vdash \mathit{all}(s, D-r)\wedge \mathit{not\ all}(t, D-r) \rightarrow \mathit{not\ all}(t, s)$ (i.e. *AOO-2*, by [27], F5 and F6)
[29] $\vdash \mathit{some}(r, t)\wedge \mathit{no}\neg(t, s) \rightarrow \mathit{not\ all}\neg(r, s)$ (by [6], F1 and F3)
[30] $\vdash \mathit{some}(r, t)\wedge \mathit{no}(t, D-s) \rightarrow \mathit{not\ all}(r, D-s)$ (i.e. *EIO-1*, by [29] and D3)
[31] $\vdash \mathit{some}(t, r)\wedge \mathit{no}(t, D-s) \rightarrow \mathit{not\ all}(r, D-s)$ (i.e. *EIO-3*, by [30] and F9)
[32] $\vdash \mathit{some}(t, r)\wedge \mathit{no}(D-s, t) \rightarrow \mathit{not\ all}(r, D-s)$ (i.e. *EIO-4*, by [31] and F10)
[33] $\vdash \mathit{some}(r, t)\wedge \mathit{no}(D-s, t) \rightarrow \mathit{not\ all}(r, D-s)$ (i.e. *EIO-2*, by [30] and F10)

So far, on the basis of 33 reasoning steps, Theorem 2 has completed the task of transforming the syllogism *AEE-2* to the other 23 valid syllogisms.

4. Conclusion

This paper firstly formalizes Aristotelian syllogisms based on the tripartite structure of categorical propositions, and then uses the truth definitions of categorical propositions to prove the validity of the Aristotelian syllogism *AEE-2*. Then, the remaining 23 valid syllogisms are derived from the syllogism *AEE-2* with the help of relevant facts, inner and outer negation definitions of quantifiers, and deductive rules. In other words, this paper reveals the reducible relationship between/among these 24 syllogisms and establishes a succinct formal reason system for Aristotelian syllogistic.

The deductive reasoning not only ensures consistency in its results, but also provides a concise mathematical paradigm for other types of syllogisms (such as generalized modal syllogisms, syllogisms with adjectives). How to implement this formal method on a computer? This question requires further study.

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